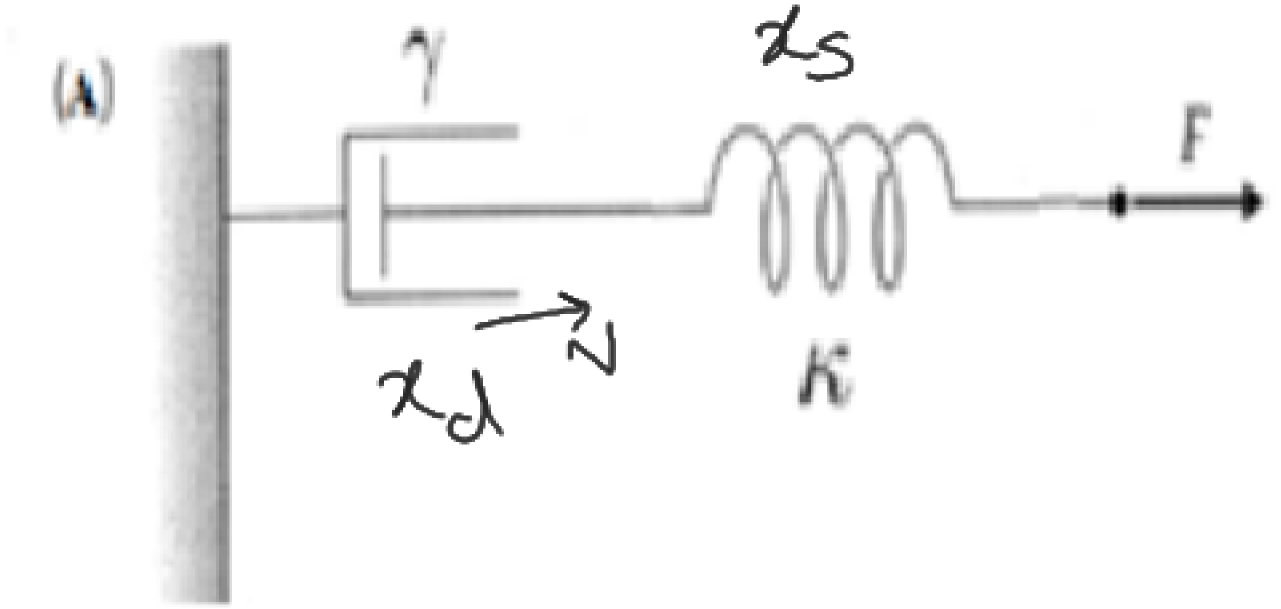
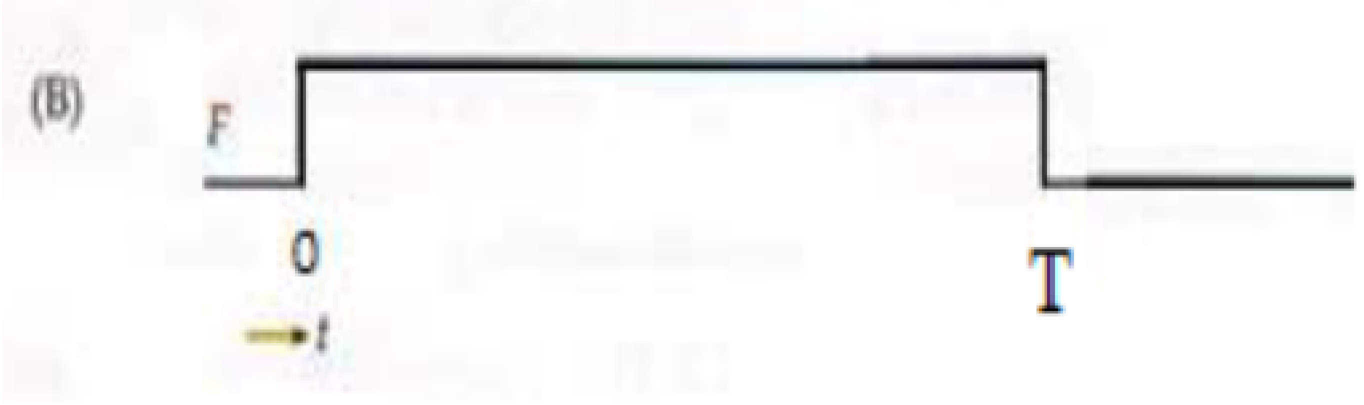
1. Consider a system consisting of a spring and a dashpot in series as shown below in (A). The stiffness or spring constant of the spring is k and drag coefficient of dashpots is γ. Initially spring and both dashpots are at rest. Suppose a constant force F is abruptly applied to this system as at t=0 and is maintained for t = T as shown below in (B), and force is then abruptly removed (i.e. F=0 for t > T ), where T is an arbitrary time. Find out the expression for displacement x(t) of the system for t < T, t = T and for t > T?




$kx_s = F$

$\Rightarrow x_s = \frac{F}{k}$

$\gamma v = F \quad (t<T)$

$v = \frac{dx_d}{dt} = \frac{F}{\gamma}$

$\int_0^{x_d} dx_d = \int_0^{t} \frac{F}{\gamma}\, dt \Rightarrow x_d = \frac{Ft}{\gamma}$

$x = x_s + x_d = \boxed{\frac{F}{k} + \frac{Ft}{\gamma} \quad (t<T)}$

At $t=T$,

$\boxed{x = \frac{F}{k} + \frac{FT}{\gamma}}$

For $t>T$, $F=0$

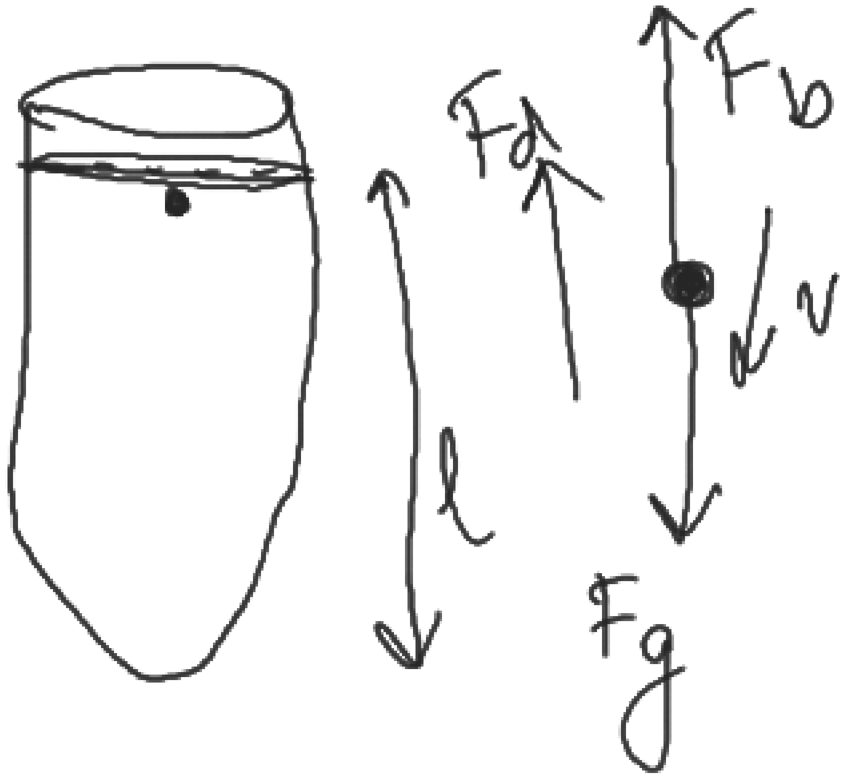
$kx_s = 0 \Rightarrow x_s = 0$

$\gamma v = 0 \Rightarrow \frac{dx_d}{dt} = 0$

$\Rightarrow x_d = \text{const} = \frac{FT}{\gamma}$

$x = x_s + x_d = \boxed{\frac{FT}{\gamma} \quad (t>T)}$

**2.** Consider the sedimentation of a spherical bio-molecule of radius 1 nm, initially right below the surface, in an Eppendorf tube of length 2 centimeters filled with water. Suppose that density of this biomolecule is ten times that of water and this bio-molecule sediments under the effect of gravity. Further assume that this bio-molecule attains a constant velocity as soon as it starts to descend in the Eppendorf tube. Calculate the sedimentation time for this bio-molecule (Density water=1000 Kg m$^{-3}$ and g=10m/s$^2$)?




Small molecule

$\Rightarrow$ no acceleration

$\Rightarrow F_d + F_b = F_g$

$F_d = \gamma v$, $F_g = mg$, $F_b = \rho_w V g$

$$F_d = F_g - F_b$$

$$\Rightarrow \gamma v = \rho_m V g - \rho_w V g$$

$$= 10\rho_w V g - \rho_w V g$$

$$= 9\rho_w V g$$

$$= 9\rho_w \times \frac{4}{3}\pi r^3 \times g$$

$$= 12\rho_w \pi r^3 g$$

$$\Rightarrow v = \frac{12\rho_w \pi r^3 g}{6\pi\eta r} = 2\rho_w r^2 g$$

$$t = l/v \longrightarrow \text{approx } \boxed{31.71 \text{ years}}$$

3. A micron size cylindrical rod as shown below is moving with constant velocity $\vec{v}$ in x-z plane such that the angle between the axis of cylinder and velocity vector is $45^0$. Consider the motion is dominated by viscous forces. What would be the angle between the direction of net drag force on the cylinder and axis of the cylinder, if drag coefficient for motion perpendicular to axis of cylinder is $\sqrt{3}$ times that of drag coefficient for the motion parallel to axis of cylinder? (Hint: Resolve velocity vector along the axis of cylinder and parallel to axis of cylinder)

$\gamma_\perp = \sqrt{3}\,\gamma_{\parallel}$ (given)

$V_{\parallel} = V\cos 45^\circ = V/\sqrt{2}$

$V_\perp = V\sin 45^\circ = V/\sqrt{2}$

$\Rightarrow V_{\parallel} = V_\perp$

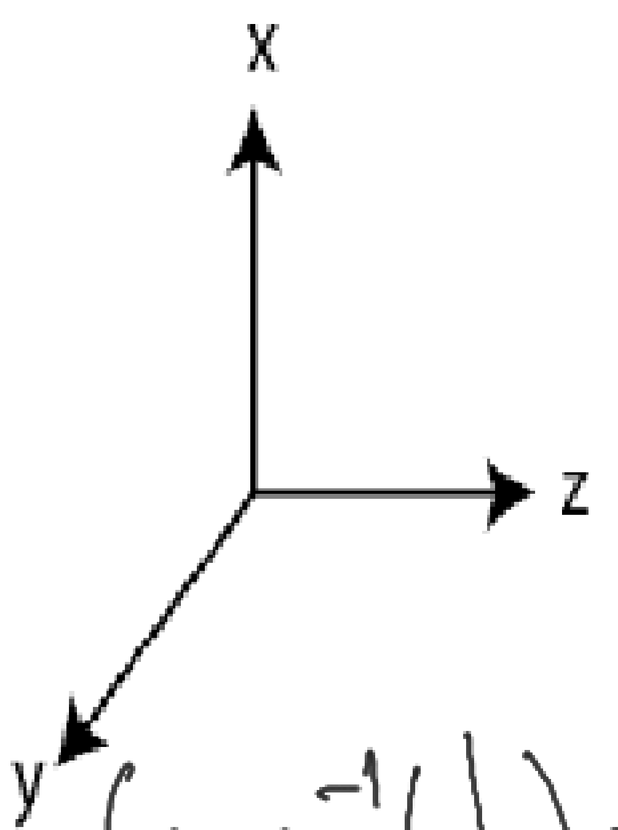
$$\boxed{F = \gamma V}$$

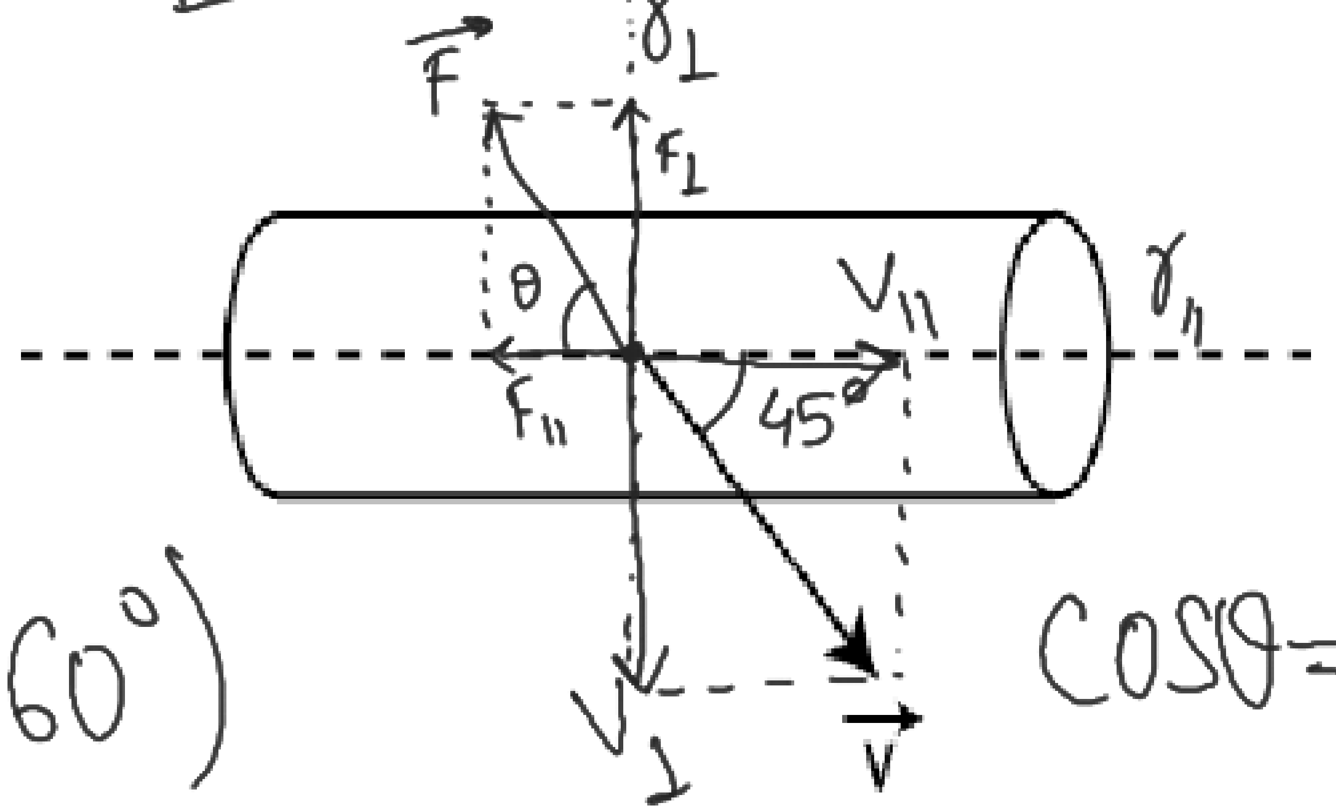
$F_{\parallel} = \gamma_{\parallel} V_{\parallel}$ $\quad F_\perp = \gamma_\perp V_\perp$

$F^2 = F_{\parallel}^2 + F_\perp^2 = \gamma_{\parallel}^2 V_{\parallel}^2 + \gamma_\perp^2 V_\perp^2$

$= \gamma_{\parallel}^2 V_{\parallel}^2 + 3\gamma_{\parallel}^2 V_{\parallel}^2 = 4\gamma_{\parallel}^2 V_{\parallel}^2$

$\Rightarrow F = 2\gamma_{\parallel} V_{\parallel}$




$\left(\cos^{-1}\left(\frac{1}{2}\right) = 60^\circ\right)$

$\cos\theta = \frac{F_{\parallel}}{F} = \frac{\cancel{\gamma_{\parallel}}\cancel{V_{\parallel}}}{2\cancel{\gamma_{\parallel}}\cancel{V_{\parallel}}} = \frac{1}{2} \Rightarrow \boxed{\theta = 60^\circ}$

**4.** Bacterial use flagella motor to rotate the helical flagellum to generate the propulsive force. Provide a qualitative explanation (using the result from Problem 3 of Tutorial 1) that if a thin, rigid helical rod as shown below is cranked about its helix axis at a certain angular speed then it can generate a net force propulsive force.

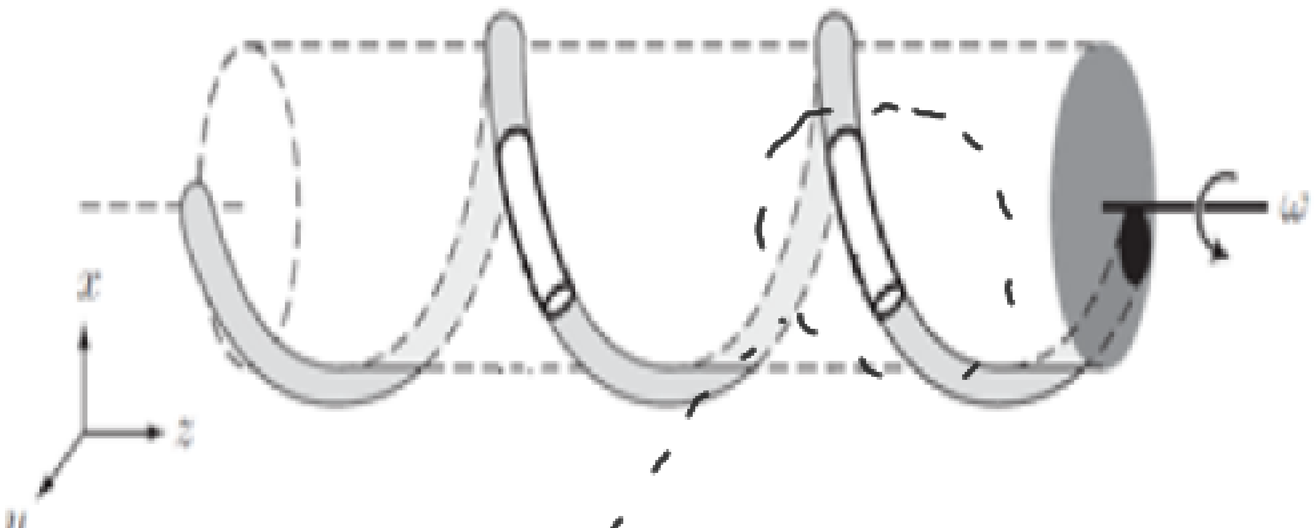
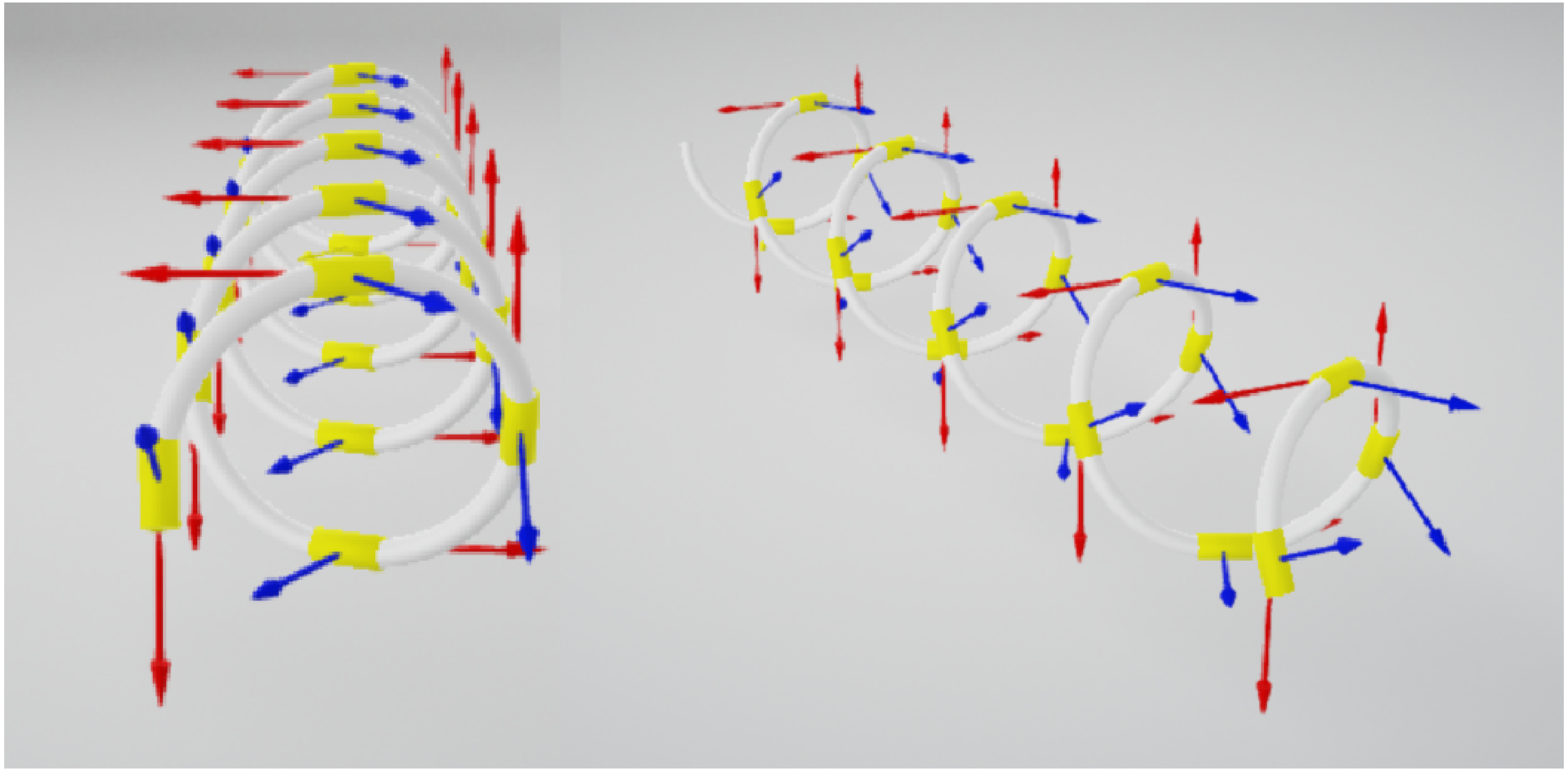
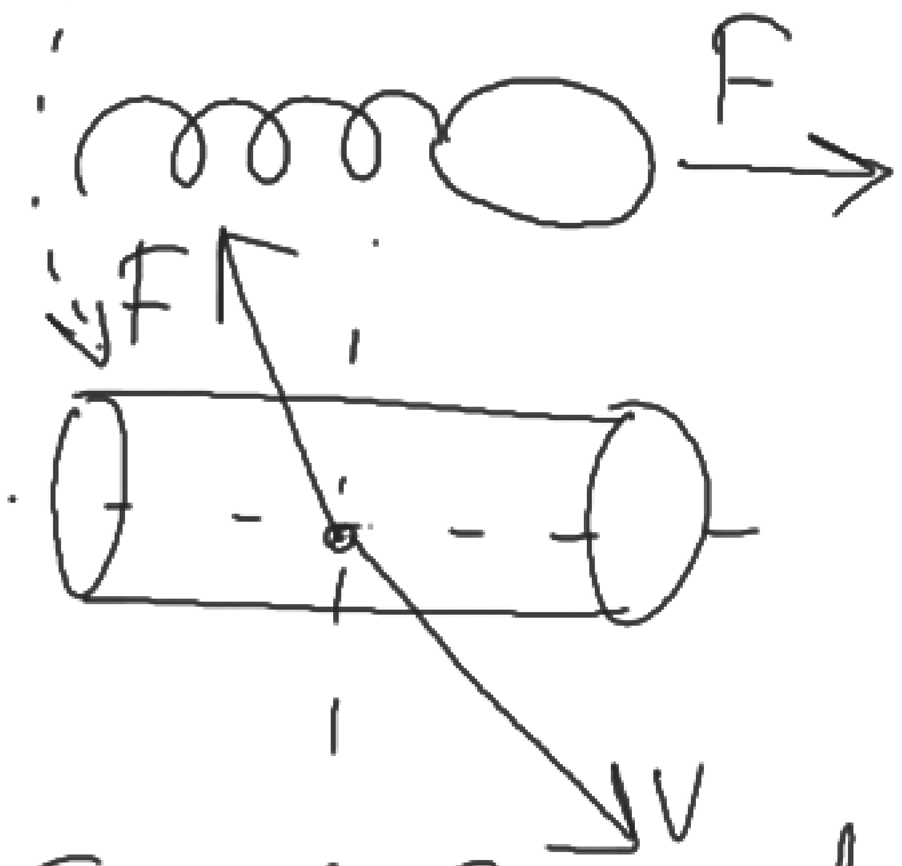



Figure Source: Biological Physics, Philip Nelson (Publisher: W. H. Freeman)






$F_x$ and $F_y$ cancel
$F_z$ add up to make it move